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w_∞ 3-algebra

Shankhadeep Chakrabortty, Alok Kumar and Sachin Jain

Institute of Physics,
Bhubaneswar 751 005, India
E-mail: sankha@iopb.res.in, kumar@iopb.res.in, sachjain@iopb.res.in

Abstract: We use 'lone-star' product of the W_∞ generators as well as their commutation relations to obtain a w_∞ 3-algebra by applying appropriate double scaling limits on the generators. We show explicitly that "Fundamental Identity condition" (FI) of 3-algebra is satisfied.

Keywords: M-Theory, Conformal and W Symmetry.

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3-Algebra relations have played an important role in the construction of the worldvolume theories of multiple $M2$ branes [1 − 3], which has attracted a great deal of attention. Of particular interest are the developments in the realization of this structure in terms of three dimensional $N = 6$ Chern-Simons theories [4]. These pioneering investigations have also generated renewed interest in the analysis of 3-algebras [5, 6] in general. In particular, several authors [7 − 10] have considered their affine extensions by showing the existence of Kac-Moody and (centerless) Virasoro 3-algebras and demonstrating some of their applications to the Bagger-Lambert theory.

Motivated by the above recent works, specially those in [7 − 10], as well as by the fact that 3-algebra relations are a rarity [3], in this paper, we explicitly obtain a classical w_∞ 3-algebra and show that our relation satisfies the 3-algebra "Fundamental Identity condition"(FI). Our construction is based on earlier work on W_∞ and $W_{1+\infty}$ symmetries (see [11, 12] *and references therein*). Using the 'lone-star' product of $W_{1+\infty}$ generators and their commutation relations we write down a 3-algebra relation. The structure constants for such a 3-algebra relation simplify in a double scaling limit and we show the validity of the FI through direct verification presented below. We also show that the 4-brackets among the resulting w_∞ generators vanish.

We now start with the commutation relations defining $W_{1+\infty}$ algebra written in terms of generators $\tilde{V}^i_m$ [11]:

$$[\tilde{V}^i_m, \tilde{V}^j_n] = \sum_{l\geq 0} q^{2l} \tilde{g}^{ij}_{2l}(m,n) \tilde{V}^{i+j-2l}_{m+n} + q^{2i} \tilde{c}_i(m) \delta^{ij} \delta_{m+n,0}, \tag{1}$$

where superscripts i, j, l, representing the conformal spin of the generators, are in general integers: $-1, 0, 1, \ldots$ etc. whereas integer subscripts m, n can take arbitrary positive or negative values. We also have:

$$\tilde{g}^{ij}_l(m,n) \equiv g^{ij}_l\left(m, n, -\frac{1}{2}\right) \tag{2}$$

given by an expression:

$$g^{ij}_l(m,n,s) = \frac{1}{2(l+1)!} \phi^{ij}_l(s) N^{ij}_l(m,n). \tag{3}$$

Explicitly, $\phi^{ij}_l(s)$ are given by a generalized hypergeometric function:

$$\phi^{ij}_l(s) = {}_4F_3 \left[\begin{matrix} -\frac{1}{2} - 2s, \ \frac{3}{2} + 2s, & -\frac{l}{2} - \frac{1}{2}, & -\frac{l}{2} \\ & & & ;1 \\ -i - \frac{1}{2}, \ -j - \frac{1}{2}, & i + j - l + \frac{5}{2} & \end{matrix} \right] \tag{4}$$

and

$$N^{ij}_l(m,n) = \sum_{k=0}^{l+1} (-1)^k \begin{pmatrix} l+1 \\ k \end{pmatrix} (2i+2-l)_k [2j+2-k]_{l+1-k} [i+1+m]_{l+1-k} [j+1+n]_k, \tag{5}$$

where $[a]_n$ stands for $\frac{a!}{(a-n)!}$ and $(a)_n$ stands for $\frac{(a+n-1)!}{(a-1)!}$. Also, in eq. (1) q is an arbitrary scaling parameter, which we will fix later on through a double scaling process. Finally, the central term of the algebra in eq. (1) can be consistently set to zero and corresponds to the analysis of classical symmetries.

Another property of interest for us will be the 'lone-star' product of the $W_{1+\infty}$ generators:

$$\tilde{V}^i_m \star \tilde{V}^j_n = \sum_{l\geq -1} q^l \tilde{g}^{ij}_l(m,n)\tilde{V}^{i+j-l}_{m+n}. \tag{6}$$

This star product is classical, since it does not contain information about the central term. As in the following we make use of the relation (6) to construct our 3-algebra, this analysis therefore holds for the classical case only. Note also that the commutation relation (1) follows from the 'lone-star' product eq. (6) (in absence of central term) by realizing that coefficients $\tilde{g}^{ij}_l(m,n)$ are symmetric under the simultaneous interchange of i,j and m,n for odd l's whereas they are antisymmetric for even l's. We now restrict ourselves to the case when the central term is absent.

Now, using the definition of the 3-algebra relation:

$$[A,B,C] = A[B,C] + B[C,A] + C[A,B] \tag{7}$$

and the commutation relation (1) as well as the star product (6), we can write the 3-algebra relation:

$$\begin{aligned}[\tilde{V}^a_m, \tilde{V}^b_n, \tilde{V}^c_p] = \sum_{l\geq 0, r\geq -1} & q^{2l+r}[\tilde{g}^{b,c}_{2l}(n,p)\tilde{g}^{a,b+c-2l}_r(m,n+p) \\ &+\tilde{g}^{c,a}_{2l}(p,m)\tilde{g}^{b,c+a-2l}_r(n,m+p)+\tilde{g}^{a,b}_{2l}(m,n)\tilde{g}^{c,a+b-2l}_r(p,m+n)]\tilde{V}^{a+b+c-2l-r}_{m+n+p},\end{aligned} \tag{8}$$

where the index r, for a given l, runs over indices $r = -1, 0, \ldots, (a+b+c-2l+1)$, whereas the running of index l in the three terms in rhs of eq. (8) is from zero upto $b+c$, $c+a$ and $a+b$ respectively.

The 3-algebra relation in eq. (8) may be of interest in its own right, however in the following we present a simpler situation by using a double scaling limit on the above relation. We also recall that a similar procedure (but with a single scaling parameter q) was used earlier to obtain the w_∞-algebra from W_∞. Relationship between w_∞-algebra and area preserving reparameterizations of 2-torus are also well known [13]. In this paper we will observe an interesting relation at the 3-algebra level by comparing the structure constants of the 3-algebra emerging from the 3-bracket given in eq. (8), after taking the double scaling limit, with the one for the classical Nambu 3-brackets of globally defined functions[1] f, g, h on T^2.

Now, to apply our double scaling, we scale all the generators $\tilde{V}^a_m$ in eq. (8) by a parameter β. Note that such a scaling is in addition to the one given in [11] which lead to the powers of q^{2l} in the commutation relation (1). We also note that the smallest power of q in eq. (8) corresponds to $l=0$ and $r=-1$. In order to keep only this term, after the double scaling, we take the limits: $q\to 0$, $\beta\to\infty$ such that $\beta^2 q = 1$. We then obtain the simplified 3-algebra in terms of the rescaled generators w^a_m's:

$$[w^a_m, w^b_n, w^c_p] = [c(n-m) + b(m-p) + a(p-n)]w^{a+b+c+1}_{m+n+p}, \tag{9}$$

[1]This point was communicated to us by Cosmos Zachos and collaborators.

where we have also made use of the fact that

$$\tilde{g}_{-1}^{ab}(m,n) = 1, \quad \tilde{g}_0^{ab}(m,n) = (b+1)m - (a+1)n. \tag{10}$$

We now verify that w_∞ 3-algebra satisfies the FI, written in the present case as:

$$\begin{aligned}[][w^a_m, w^b_n, [w^c_p, w^d_q, w^e_r]] = [[w^a_m, w^b_n, w^c_p], w^d_q, w^e_r]] + [w^c_p, [w^a_m, w^b_n, w^d_q], w^e_r]] \\ +[w^c_p, w^d_q, [w^a_m, w^b_n, w^e_r]].\end{aligned} \tag{11}$$

A discussion on the necessity of the FI's defining the Leibniz rule for the action of 3-brackets, as well as an analysis of the associativity constraints in such cases, is presented in [8]. In our case, evaluating the four terms we obtain:

$$\begin{aligned}[][w^a_m, w^b_n, [w^c_p, w^d_q, w^e_r]] &= [e(q-p) + d(p-r) + c(r-q)] \times \\ &\quad [(c+d+e+1)(n-m) + b(m-p-q-r) \\ &\qquad + a(p+q+r-n)] w^{a+b+c+d+e+2}_{m+n+p+q+r}, \end{aligned} \tag{12}$$

$$\begin{aligned}[][[w^a_m, w^b_n, w^c_p], w^d_q, w^e_r]] &= [c(n-m) + b(m-p) + a(p-n)] \times \\ &\quad [e(q-m-n-p) + d(m+n+p-r) \\ &\qquad + (a+b+c+1)(r-q)] w^{a+b+c+d+e+2}_{m+n+p+q+r}, \end{aligned} \tag{13}$$

$$\begin{aligned}[][w^c_p, [w^a_m, w^b_n, w^d_q], w^e_r]] &= [d(n-m) + b(m-q) + a(q-n)] \times \\ &\quad [e(m+n+q-p) + (a+b+d+1)(p-r) \\ &\qquad + c(r-m-n-q)] w^{a+b+c+d+e+2}_{m+n+p+q+r}, \end{aligned} \tag{14}$$

$$\begin{aligned}[][w^c_p, w^d_q, [w^a_m, w^b_n, w^e_r]] &= [e(n-m) + b(m-r) + a(r-n)] \times \\ &\quad [(a+b+e+1)(q-p) + d(p-m-n-r) \\ &\qquad + c(m+n+r-q)] w^{a+b+c+d+e+2}_{m+n+p+q+r}. \end{aligned} \tag{15}$$

Using eqs. (12), (13), (14) and (15), it can now be checked directly that the 3-algebra in eq. (9) satisfies the FI in eq. (11).

We have therefore obtained a 3-algebra generalization of the w_∞-algebra. Note that our double scaling is such that it gives a nontrivial 3-algebra in terms of w_∞ generators. This double scaling would however make the original commutation relations [11] of w_∞ generators trivial. There is, however, no inconsistency with our analysis above, since the 'lone-star' product also goes to infinity in this limit, thus giving us a well defined 3-algebra with finite coefficients. We have also analyzed the expressions for the (totally antisymmetrized) 4-brackets involving the generators w^a_m, using the relation (9):

$$[w^a_m, w^b_n, w^c_p, w^d_q] = w^a_m[w^b_n, w^c_p, w^d_q] - w^b_n[w^c_p, w^d_q, w^a_m] + w^c_p[w^d_q, w^a_m, w^b_n] - w^d_q[w^a_m, w^b_n, w^c_p]. \tag{16}$$

By explicit calculation we find that it is identically zero, a result similar to the one [8] for the Virasoro 3-algebra.

As already pointed out before, above results can also be reinterpreted in terms of the algebraic structure of the reparameterizations of 2-torus through the evaluation of the classical Nambu 3-brackets (3CNB) of globally defined functions f, g, h on a 2-torus. 3CNB

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of functions f, g, h, that are completely antisymmetrized, are defined as the Jacobian of the transformation from (x, y, z) to $(f(x, y, z), g(x, y, z), h(x, y, z))$:

$$\begin{aligned}\{f, g, h\} \equiv \frac{\partial(f, g, h)}{\partial(x, y, z)} &\equiv\; f\{g, h\} + g\{h, f\} + h\{f, g\} \\ &= \frac{\partial f}{\partial x}\left(\frac{\partial g}{\partial y}\frac{\partial h}{\partial z} - \frac{\partial h}{\partial y}\frac{\partial g}{\partial z}\right) + \frac{\partial g}{\partial x}\left(\frac{\partial h}{\partial y}\frac{\partial f}{\partial z} - \frac{\partial f}{\partial y}\frac{\partial h}{\partial z}\right) + \frac{\partial h}{\partial x}\left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial z} - \frac{\partial g}{\partial y}\frac{\partial f}{\partial z}\right).\end{aligned} \tag{17}$$

Now, to establish the connection with our results given above, we note that by choosing:

$$\begin{aligned} f \equiv w^a_m &= \sqrt{z}\exp\left(\left(a + \frac{1}{2}\right)x + my\right), \\ g \equiv w^b_n &= \sqrt{z}\exp\left(\left(b + \frac{1}{2}\right)x + ny\right), \\ h \equiv w^c_p &= \sqrt{z}\exp\left(\left(c + \frac{1}{2}\right)x + py\right), \end{aligned} \tag{18}$$

we obtain the 3CNB of generators $\{w^a_m, w^b_n, w^c_p\}$, which matches with the 3-bracket given in eq. (9) (by a constant scaling of the generators), with structure constant:

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ m & n & p \end{vmatrix}. \tag{19}$$

We also note that a somewhat similar structure appeared in the Moyal (sine) brackets of [13] and its correspondence to 3-algebra structure constants in our case will be of interest to examine. Also, it is noticed from eq. (18) that the 3-algebra generators of eq. (9) can be identified with the modes of the deformations of 2-torus [13]. In the present case, however, one also needs to multiply the exponential functions in eqs. (18) by an extra factor $\sqrt{z}$ common to all three generators in the 3CNB. The geometric interpretation of such an extra factor may be possible by identifying the complete geometry as a direct product of 2-torus with a point, since the deformation mode along the z direction is frozen.

We now comment on the the validity of the 3-bracket expression (before taking the scaling limit), i.e. eq. (8), as a proper 3-algebra relation. First of all it is interesting to note that all the terms which are even in r, in the sum in the rhs of eq. (8), vanish. This follows from an observation of the Jacobi identity involving the $W_{1+\infty}$ generators $\tilde{V}^a_m$. Moreover, we have explicitly analyzed the FI for some of the low lying (i.e. in indices a, b) generators $\tilde{V}^a_m$ and our results imply that w_∞ does not extend to the full $W_{1+\infty}$. In other words, not all 3-brackets satisfy the FI, a situation already known for the case of compact Lie group generators. Eq. (8) may, however, still be of interest in obtaining a consistent higher bracket.

It is also of interest to generalize this result in several other directions, such as in obtaining 'primary' field representations, supersymmetric generalizations etc. . . These topics are currently under investigation.

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